4. G. T. Sergeev, "Internal heat and mass transfer with gas filtration through a porous wall and the presence of chemical reactions," Izv. Akad. Nauk BSSR, Ser. Fiz.-Ener. Nauk, No. 2 (1975).
5. M. D. Mikhailov, "Stationary temperature for porous cooling," Inzh.-Fiz. Zh., 11, No. 2 (1966).
6. G. T. Sergeev, Principles of Heat and Mass Transfer in Reacting Systems [in Russian], Nauka i Tekhnika, Minsk (1976).
7. O. N. Bryukhanov, Radiation-convective Heat Transfer during Gas Combustion in Perforated Systems [in Russian], Leningrad State Univ. Press, Leningrad (1977).

OPTIMIZATION OF CONVECTIVE CIRCULAR FINS

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The volume of a circular fin whose thickness is inversely proportional to the square of the radius is optimized.

The books [1, 2] provide an idea of the present state of the theory and practical application of finned heating surfaces. These books also examine the question of optimizing the volume of the fins. The object of optimization is to select a fin with minimum volume for transferring a specified amount of heat under known thermophysical conditions. Whereas for straight fins the problem of optimization is solved by several variants of the cross section of the fin, for circular fins only the results of [3] for fins of constant thickness are given.

We note that for hyperbolic profiles examined in [1, Tables 1-5], the problem of optimization is solved very simply in the case of the thickness of the fin being inversely proportional to the square of the radius. If we use the notation of [1], this dependence has the form

$$\delta/\delta_1 = R^{-2}, \tag{1}$$

$$R = r/r_1. \tag{2}$$

For convenience, we denote the height of the fin

$$h = r_2 - r_1, \tag{3}$$

and the parameter of the fin N is expressed in the form

$$N^2 = 2\alpha h^2/\lambda\delta_1. \tag{4}$$

To make the circular rib more comparable with a straight rib, we refer the thermal flux and the volume of the circular rib to a unit length of the base

$$Q_1 = Q_0/2\pi r_1 = \alpha\vartheta_1\eta h(R_2 + 1), \tag{5}$$

$$V_1 = \frac{1}{2\pi r_1}\int_{r_1}^{r_2} 2\pi r\delta dr = h\delta_1 \ln R_2/(R_2 - 1). \tag{6}$$

Determining the value of δ_1/h^2 from (4), and h from (5), we can express the product in (6) as

$$h\delta_1 = (\delta_1/h^2)(h^3), \tag{7}$$

and formula (6) is transformed to the form

$$V_1 = \left(\frac{Q_1}{\alpha\vartheta_1}\right)^3 \frac{2\alpha}{\lambda N^2} \frac{\ln R_2}{\eta^3(R_2 + 1)^3(R_2 - 1)}. \tag{8}$$

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For further formal simplification we take

$$x = (R_2 + 1)N/2, \tag{9}$$

$$V_0 = Q_1^3/2\alpha^2\lambda\vartheta_1^3, \tag{10}$$

and according to formulas (1)-(79) in [1],

$$\eta = \operatorname{th} x/x. \tag{11}$$

Thus the dimensionless volume of the fin is expressed as

$$v = V_1/V_0 = \frac{\ln R_2}{(R_2^2 - 1)}\,\frac{x}{\operatorname{th}^3 x}\,. \tag{12}$$

The problem of optimizing the fin with specified ratio of the radii R_2 consists in finding the minimum value of v, i.e.,

$$\frac{\partial v}{\partial x} = 0. \tag{13}$$

Substitution of (12) into (13) leads to an equation whose solution yields the following optimum parameters of the fin:

$$\begin{aligned} x_{\text{opt}} &= 1.4192, \\ \eta_{\text{opt}} &= 0.6267, \\ N_{\text{opt}} &= 2.8384/(R_2 + 1), \\ v_{\text{opt}} &= 2.017 \ln R_2/(R_2^2 - 1). \end{aligned} \tag{14}$$

It should be noted that a circular fin with profile (1) has the following properties.

1. It follows from (11) that the heat exchange efficiency is determined from graphs and formulas for straight fins of constant thickness if the parameter N in them is replaced by x from formula (9) [1, p. 15; 2, pp. 80, 85; 4, p. 266]).

2. The optimum heat exchange efficiency does not depend on the ratio of the radii and is constant ($\eta_{\text{opt}} = 0.6267$) for any arbitrary R_2.

NOTATION

h, height of the fin; N, parameter of the fin (4); Q_0, heat flux of the fin, W; Q_1, heat flux per 1 m of the fin base (5), W/m; R, dimensionless radius (2); r, radius; V_0, volume (10), m^2; V_1, volume of fin per 1 m of base, m^2; v, dimensionless volume (12); x, parameter (9); α, heat transfer coefficient; δ, thickness of the rib; η, heat exchange efficiency of the rib; ϑ, excess temperature relative to the ambient temperature; λ, thermal conductivity. Subscripts: 1, base of the rib; 2, outer edge of the rib; opt, optimized value.

LITERATURE CITED

1. L. I. Roizen and I. N. Dul'kin, Thermal Calculation of Ribbed Surfaces [in Russian], Énergiya, Moscow (1977).
2. D. Kern and A. Kraus, Extended Surface Heat Transfer, McGraw-Hill (1972).
3. A. Brown, Int. J. Heat Mass Transfer, 8, 655-662 (1965).
4. Thermal Calculation of Boiler Assemblies (Normative Method) [in Russian], 2nd rev. ed., Énergiya, Moscow (1973).